ECE 333 – Renewable Energy Systems

5. Wind Power

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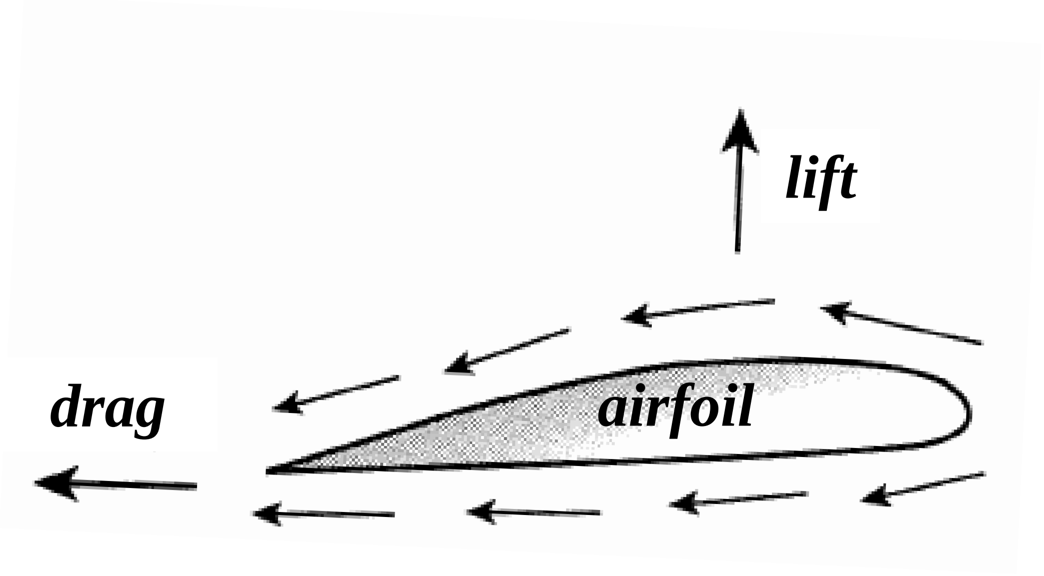
OUTLINE

- **The physics of rotors**
- **Evaluation of power in the wind**
- **The specific power definition and analysis**
- **Variation of *specific power* with temperature and altitude**
- **The impacts of tower height on wind turbine output**

ROTOR BASICS

- We provide a brief introduction to how the rotor blades extract energy from the wind
- *Bernoulli's principle* is the basis of the explanation of how an airfoil – be it an airplane wing or a wind turbine blade – obtains lift

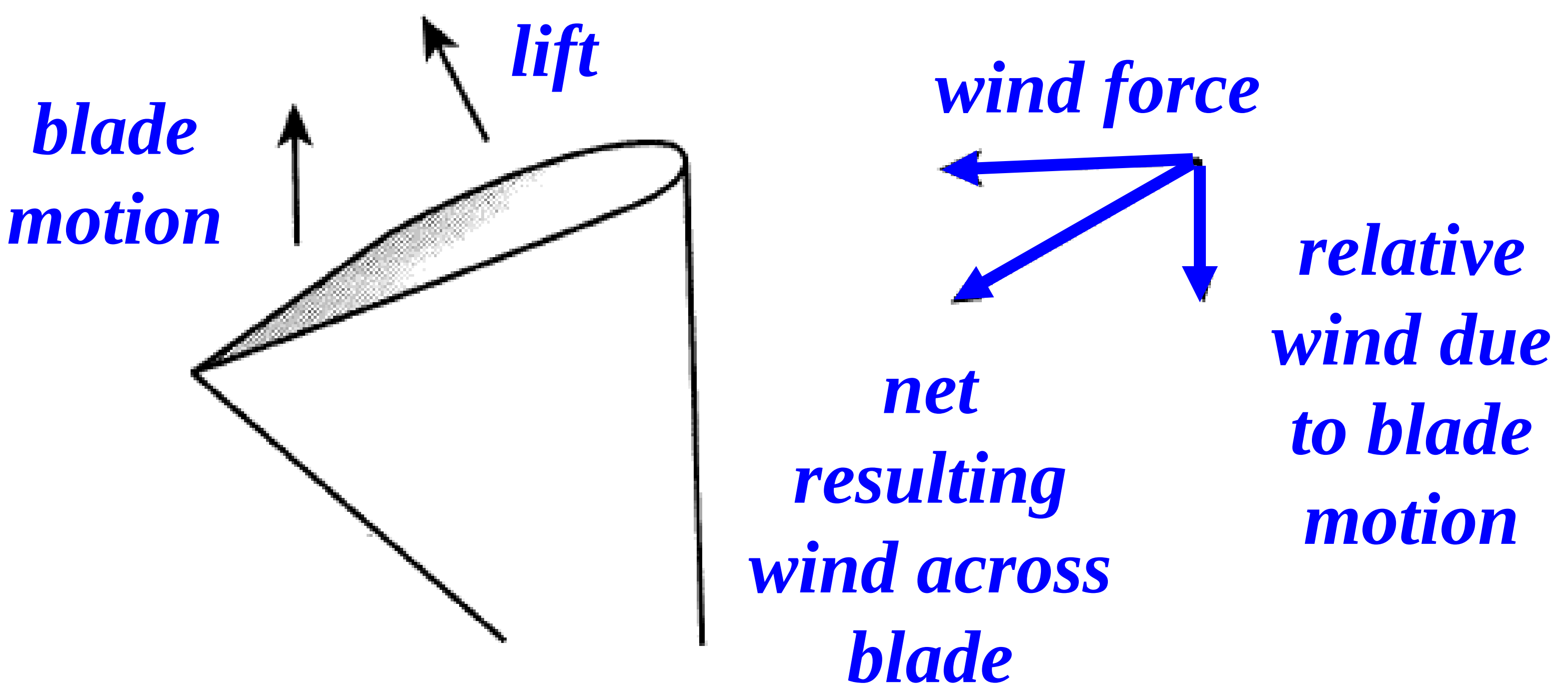


ROTOR BASICS

- air traveling over the top of the airfoil covers a longer distance before it rejoins the air using the shorter path under the foil
- air on top travels faster resulting in lower pressure than that under the airfoil
- the difference between the two pressures creates the lifting force that holds an airplane up and that rotates the wind turbine blade

ROTOR BASICS

- The situation with a rotor is more complicated than that of an airplane wing for a number of reasons:

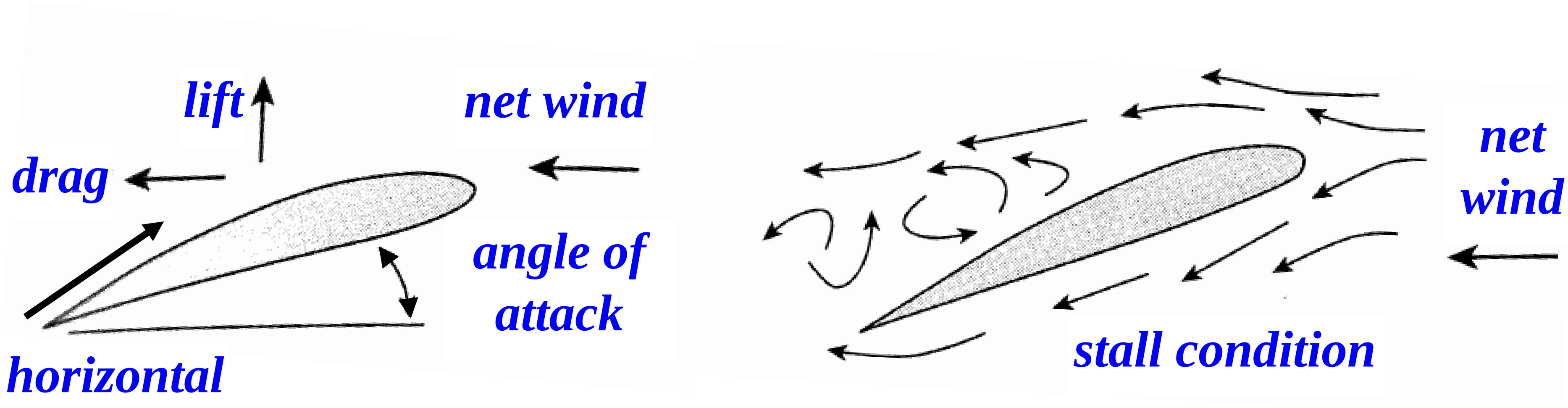


ROTOR BASICS

- a rotating blade experiences air moving toward it from the wind and from the relative motion of the blade as it spins
- the combined effect of the wind itself and the rotating blade results in a force that is at the appropriate angle so that the force is along the blade and can provide the lift that moves the rotor along

ROTOR BASICS

- as the blade speed at the tip is faster than near the hub, the blade must be twisted along its length to keep the appropriate angle



- the angle between the wind and the airfoil is referred to as the *angle of attack*

ROTOR BASICS

- **as the angle of attack increases, the *lift* increases but so does the *drag***
- **too large of an angle of attack can lead to a stall phenomenon due to the resulting turbulence**
- **wind turbines are equipped with a mechanism to shed some wind power lest the generator be damaged**

POWER IN THE WIND

- We wish to analytically characterize the level of power associated with wind
- For this purpose, we view wind as a “packet” of air with mass m moving at a constant speed v : please note, this assumption represents a major simplification since air is a fluid; however, the simplified modeling is useful to explain the key concepts

POWER IN THE WIND

- The kinetic energy of wind is

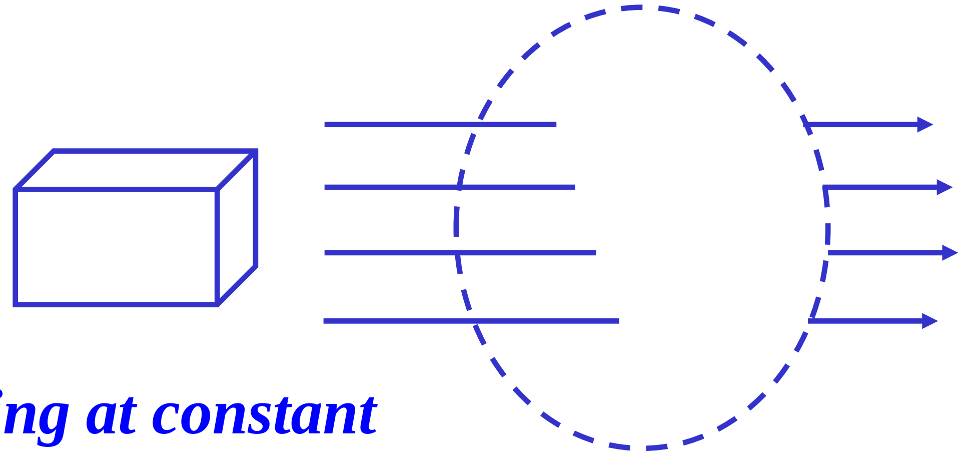
$$\varepsilon = \frac{1}{2} m v^{2}$$

- Power is simply the rate of change in energy and so we view the power in the mass of air m moving at constant speed v through area a as the rate at which the mass m passes through area a

POWER IN THE WIND

mass of air

m



moving at constant

speed v

area a

power through area a $= \dfrac{d\varepsilon}{dt}$

$$\frac{d\varepsilon}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v^{2}\right) = \frac{1}{2}\frac{dm}{dt} v^{2}$$

POWER IN THE WIND

- The term $\frac{dm}{dt}$ is the rate of flow of the mass of air through area a and is given by $\rho a v$ where ρ is the air density, i.e., the mass per unit of volume
- The volume w of mass m is given by the area a times the "length" of mass m
- Over time dt, the mass m moves a distance $v\,dt$ resulting in the volume

POWER IN THE WIND

$$dw = a\,v\,dt$$

- Now

$$\frac{dm}{dt} = \frac{dm}{dw} \bullet \frac{dw}{dt} = \frac{dm}{dw} \bullet a\,v$$

and

$$\frac{dm}{dw} = \rho \longleftarrow \text{air density}$$

- Thus the power in the wind is

$$p_w = \frac{1}{2} \rho\, a\, v^{3}$$

UNITS IN THE p_w EQUATION

- We consider the units in

$$p_w = \frac{1}{2}\rho a v^3$$

W → p_w

air density at 15° C and 1 atm → $1.225\ \frac{kg}{m^3}$ → ρ

m^2 → a

$\left(\frac{m}{s}\right)^3$ → v^3

$$\frac{kg\left(\frac{m}{s}\right)^2}{s} = \frac{J}{s}$$

UNITS IN THE p_w EQUATION

- The power in wind is, typically, expressed in units per cross sectional area – $\frac{W}{m^2}$
- We refer to the expression for p_w as *specific power* or *power density*
- We next consider p_w in more depth and analyze the impacts of temperature and altitude

ANALYSIS OF p_w

- The energy produced by a wind turbine is dependent on the power in the wind; to maximize the energy we therefore need to maximize p_w
- In the equation

$$p_w = \frac{1}{2} \rho a v^3$$

ρ is a fixed parameter which we cannot "control" but we can control the area a in the design of the wind turbine and we have some control over the wind speed in terms of the wind farm siting

ANALYSIS OF p_w

- The area a is the swept area by the turbine rotor: for a *HAWT* with a blade with diameter d

$$a = \pi \left(\frac{d}{2} \right)^2 = \frac{1}{4} \pi d^2$$

- Clearly, there are economies of scale that are associated with larger wind turbines:
  - cost of a turbine $\propto d$
  - power output of a turbine $\propto d^2$

and so the larger rotors are more cost effective

NATURE OF AIR DENSITY

- The air density ρ at $15°\ C$ and $1\ atm$ pressure at sea level is $1.225\ \frac{kg}{m^3}$, but the value changes as a function of temperature and altitude
- We know that ρ decreases as temperature increases since in a warmer day the air becomes thinner; a similar thinning of the air occurs with an increase in altitude

NATURE OF AIR DENSITY

- We need to go back to elementary chemistry and physics to determine the value of ρ for changes in temperature from $15° \ C$ and for altitudes above sea level
- The governing relation is the ideal gas law

$$\hat{p}w = nRT$$

where $\hat{p}$ is the pressure in atm, w is the volume in m^3, n is the mass in mol, T is the absolute

NATURE OF AIR DENSITY

temperature in K, and R is the *Avogadro number*, the ideal gas constant $8.2056 \times 10^{-5} m^{3} atm\ K^{-1} mol^{-1}$

- The pressure in *atm* is expressible in *SI units* since

$$1\ atm = 101.325\ kPa$$

where *Pa* is the abbreviation for the Pascal unit and

$$1\ Pa = \frac{N}{m^{2}}$$

TEMPERATURE VARIATION OF ρ

- We can restate the expression for ρ in terms of the molecular weight of the gas, denoted by $M.W.$, expressed in $\dfrac{g}{mol}$, as

$$\rho\left(\frac{kg}{m^3}\right) = \frac{n(mol)\cdot M.W.\left(\dfrac{g}{mol}\right)\cdot 10^{-3}\left(\dfrac{kg}{g}\right)}{w\left(m^3\right)}$$

- Air is the mixture of 5 gases and the associated $M.W.$ of each are given in the table below

TEMPERATURE VARIATION OF ρ

gas	fraction (%)	M.W. (g/mol)
nitrogen	78.08	28.02
oxygen	20.95	32.00
argon	0.93	39.95
CO_2	0.039	44.01
neon	0.0018	20.18

- **Thus,**

$$M.W.(air) = (0.7808)(28.02) + (0.2095)(32.00) + (0.0093)(39.95) + (0.039)(44.01) + (0.0018)(20.18)$$

$$= 28.97 \frac{g}{mol}$$

TEMPERATURE VARIATION OF ρ

- The ideal gas law for the air *M.W.* value obtains

$$\rho = \frac{\hat{p}(atm) \bullet M.W.\left(\frac{g}{mol}\right)}{RT}$$

$$= \frac{\hat{p}(atm) \cdot (28.97)\left(\frac{g}{mol}\right) \cdot 10^{-3}\left(\frac{kg}{g}\right)}{T(K) \cdot (8.2056 \times 10^{-5})\left(\frac{m^3 \cdot atm}{K \cdot mol}\right)}$$

$$\rho\left(\frac{kg}{m^3}\right) = 353.1\,\frac{\hat{p}}{T}\left(\frac{atm}{K}\right)$$

TEMPERATURE VARIATION OF ρ

- Then, at $30°\ C$ at $1\ atm$

$$\rho\left(30°\ C\right) = \frac{(353.1)(1)}{30 + 273.15} = 1.165\ \frac{kg}{m^3}$$

while at $45°\ C$ at $1\ atm$

$$\rho\left(45°\ C\right) = \frac{(353.1)(1)}{45 + 273.15} = 1.110\ \frac{kg}{m^3}$$

- Note that the doubling (tripling) of the $15°\ C$ temperature results in a $5\ \%$ ($9\ \%$) decrease in air density; these reductions, in turn translate in the same $\%$ reductions in power

ALTITUDE VARIATION OF ρ

- A change in altitude brings about a change in air pressure; we evaluate the ramification of such a change
- We consider a static column of air with cross-sectional area a and we examine a horizontal slice in that column of thickness dz and air density ρ so that its mass is $\rho\, a\, dz$

ALTITUDE VARIATION OF ρ

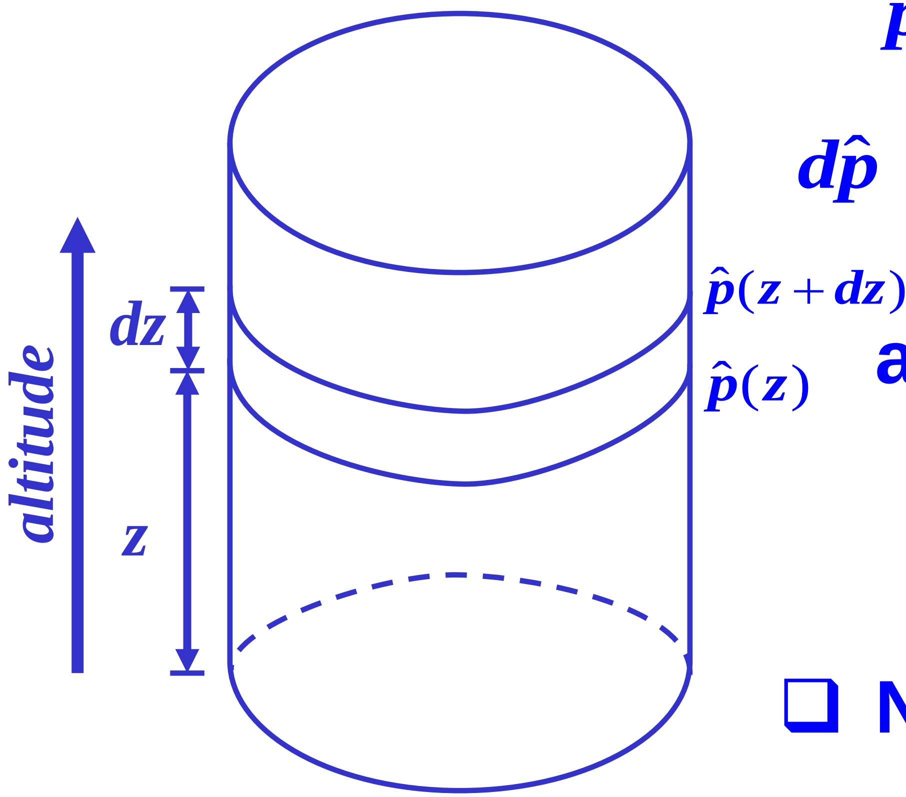
- We examine the pressures at the altitudes $z + dz$ and z due to the weight of the air above those altitudes:

$$\hat{p}(z) = \hat{p}(z + dz) + g\,\frac{\rho a\, dz}{a}$$

additional weight per unit area of the slice of thickness dz

where, $g = 9.806\,\frac{m}{s^2}$ is the gravitational constant

ALTITUDE VARIATION OF ρ



- We rewrite the difference in $\hat{p}$ at the two altitudes as

$$d\hat{p} = \hat{p}(z+dz) - \hat{p}(z) = -g\rho\, dz$$

and so

$$\frac{d\hat{p}}{dz} = -g\rho$$

- Note that

$$\rho = 353.1\,\frac{\hat{p}}{T}\left(\frac{atm}{K}\right)$$

ALTITUDE VARIATION OF ρ

- We need to make use of several conversion factors to get useable expressions

$$\frac{d\hat{p}}{dz} = -\left(\frac{353.1}{T}\right)\left(\frac{kg}{m^3}\right)\times$$

$$(9.806)\left(\frac{m}{s^2}\right)\left(\frac{1\,atm}{101.325\;Pa}\times\frac{1\,Pa}{\frac{N}{m^2}}\times\frac{1\,N}{kg\,\frac{m}{s^2}}\right)\hat{p}(atm)$$

$$= -\,0.0342\,\frac{\hat{p}}{T}$$

ALTITUDE VARIATION OF ρ

- The solution of this differential equation is complicated by the fact that the temperature also changes with altitude at the rate of $6.5° \ C$ drop for each km increase in altitude
- Under the simplifying assumption that T remains constant, the solution of the differential equation is

ALTITUDE VARIATION OF ρ

$$\hat{p}(z) = \hat{p}_0 \, exp\left(-0.0342 \frac{z}{T}\right) \qquad \hat{p}_0 = 1 \, atm$$

- It follows that

$$\rho\left(\frac{kg}{m^3}\right) = \frac{353.1}{T} \, exp\left(-0.0342 \frac{z}{T}\right)$$

where T is in K and z is in m

EXAMPLE: COMBINED TEMPERATURE AND ALTITUDE IMPACTS

- We compare the value of ρ at $25° \, C$ at $2{,}000 \, m$ to that under the standard $1 \, atm$ $15° \, C$ conditions
- We compute

$$\rho \Big|_{\substack{25° \, C \\ 2{,}000 \, m}} = \frac{353.1}{298.15} \, exp\left(-0.0342 \frac{2000}{298.15} \right) = 0.9415 \frac{kg}{m^3}$$

- The $1.225 \dfrac{kg}{m^2}$ is thus reduced by $23 \, \%$ and therefore results in a $23 \, \%$ decrease in power output – a substantial decrease

THE DEPENDENCE ON TOWER HEIGHT

- **The fact that power in the wind varies with v^3 where, v is the wind speed, implies that an increase in the wind speed has a pronounced effect on the wind output**
- **Since for a given site, v increases as the height of the tower is raised, we can generally increase the wind turbine output by mounting it on a taller tower**

THE DEPENDENCE ON TOWER HEIGHT

- A good approximation of the relationship of v and tower height h is expressed in terms of the *Hellman exponent* α – often referred to as a friction coefficient – by the relationship

$$\left(\frac{v}{v_0} \right) = \left(\frac{h}{h_0} \right)^{\alpha} ,$$

where, h_0 is the reference height with the corresponding wind speed v_0

THE DEPENDENCE ON TOWER HEIGHT

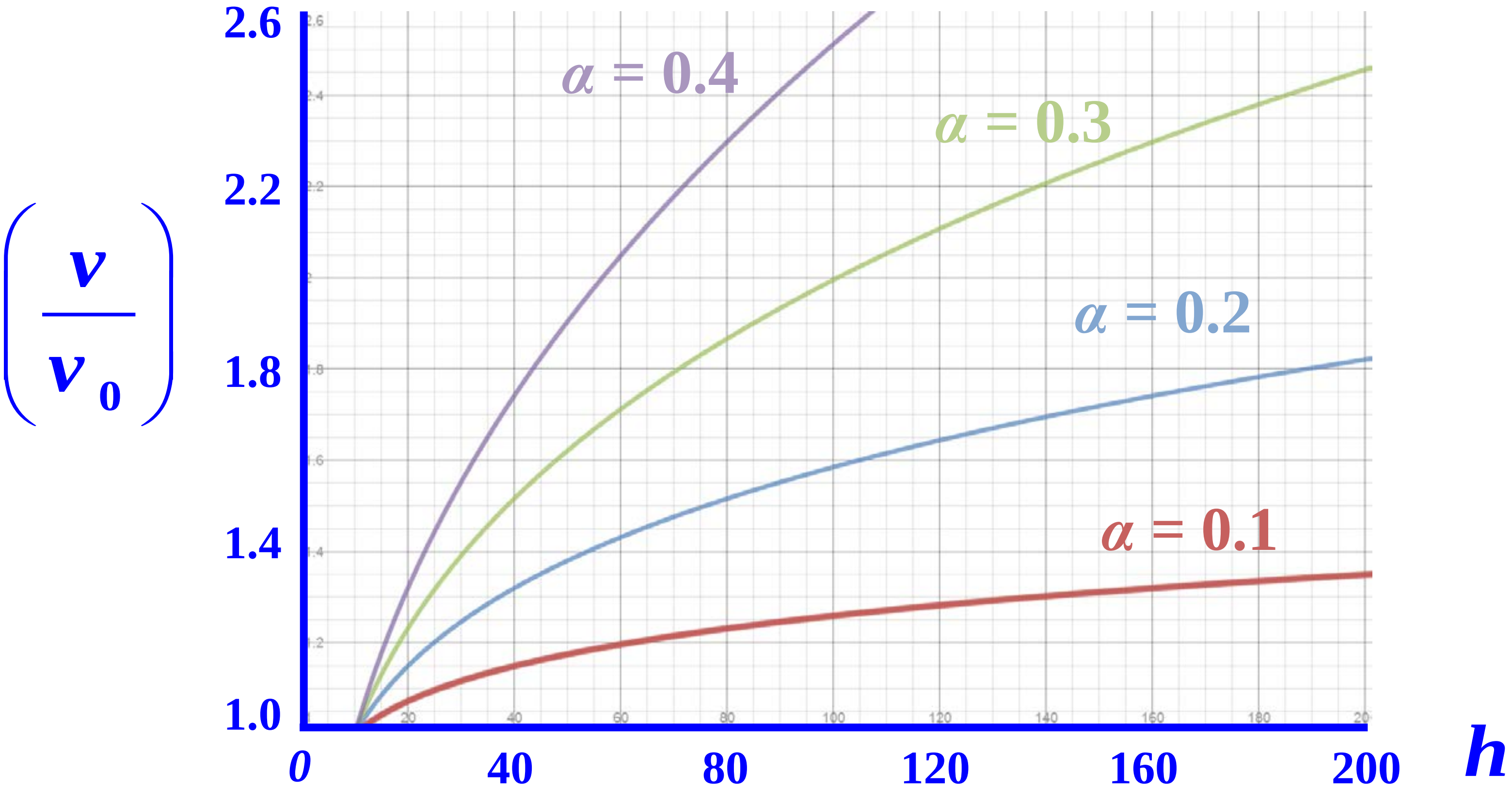
- The Hellman exponent α depends on the nature of the terrain at the site; a higher value of α indicates heavy friction – rougher terrain – and a lower value indicates low resistance faced by the wind
- Typical values for α are tabulated for different terrains

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges, and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

THE DEPENDENCE ON TOWER HEIGHT

- A typical value for h_0 is 10 m and the behavior of

  v / v_0 as a function of h / h_0 is



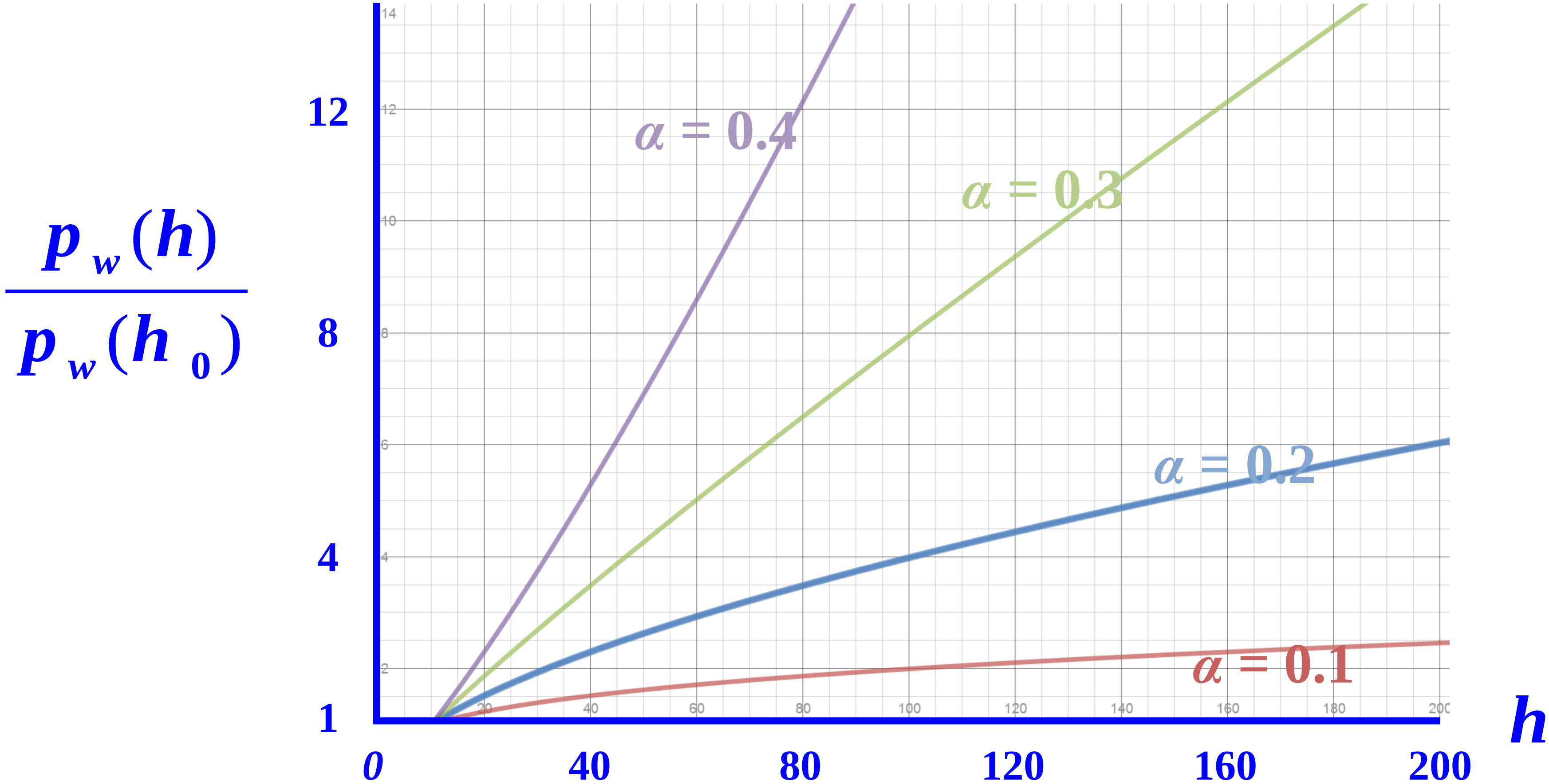
THE DEPENDENCE ON TOWER HEIGHT

- We can also determine the ratio of $p_w(h)$ to $p_w(h_0)$ under the assumption that the air density ρ remains unchanged over the range [h_0, h] using the relationship

$$\frac{p_w(h)}{p_w(h_0)} = \frac{\frac{1}{2}\rho\, a\, v^3}{\frac{1}{2}\rho\, a\, v_0^3} = \left(\frac{v}{v_0}\right)^3 = \left(\frac{h}{h_0}\right)^{3\alpha}$$

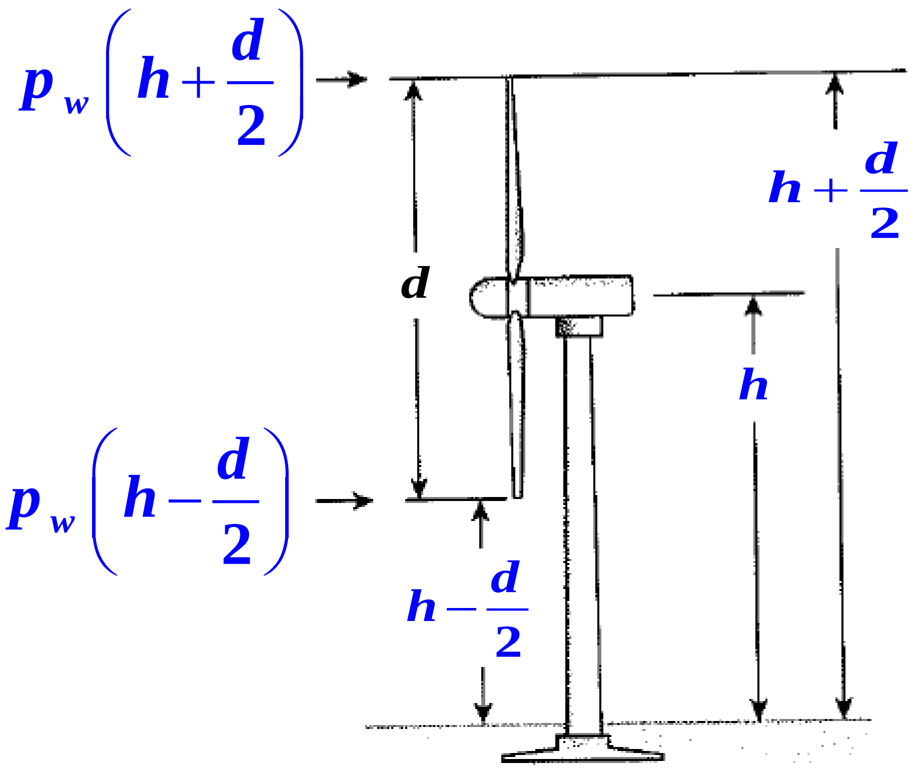
THE DEPENDENCE ON TOWER HEIGHT

- We can observe the dramatic change in the power output ratio as a function of height



THE DEPENDENCE ON TOWER HEIGHT

- A key implication of the power ratio at different heights is the fact that the stress as the turbine blade moves through an entire revolution may be rather significant, particularly over rough terrain



THE DEPENDENCE ON TOWER HEIGHT

$$p_w\left(h-\frac{d}{2}\right) \quad \textit{is the lowest value of wind output}$$

$$p_w\left(h+\frac{d}{2}\right) \quad \textit{is the highest value of wind output}$$

$$\frac{p_w\left(h+\frac{d}{2}\right)}{p_w\left(h-\frac{d}{2}\right)} = \left(\frac{h+\frac{d}{2}}{h-\frac{d}{2}}\right)^{3\alpha}$$

